An Efficient Method for Solving 3D Dielectric Planar Circuit with Parabolic Equation

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Abstract **In this paper, 3D wide-angle parabolic equation algorithm combined the Douglas scheme to the Padé series expansion is given for photonic integrated circuits and devices. The present method is easy to solve numerically by using operator splitting method in allowing wider propagation angles and the truncation error of Douglas operator scheme is fourth-order O(Δx^4) for the finite difference in the transverse direction. Therefore, it is expected that this technique improves the accuracy and efficiency of computation for optical field propagations. Finally, numerical examples are presented for a ridge-type 3D waveguide, a curved 3D waveguide model and a primitive MMI device.**

1. Introduction

Parabolic Equation Methods (called Beam Propagation Method in branches of optics [1,4,8]), in which an optical field solution can be determined by solving the one-way operator equation for the forward-propagating field, are powerful design tools for photonic integrated circuits and devices. The advantage of the FD-PEM is its simple numerical implementation and reasonable cpu-time and memory requirement. On the other hand, the method has several drawbacks because it is an approximation to the Helmholtz equation. The most serious of these is its limited angular range of principal propagation direction when dealing with 3D waveguide structures having tilted and turning waveguides. To improve the limitation on this propagation angle, a great number of PEM's have been proposed by pioneers since its inception. The most popular treatment of the square root operator is to use a high-order Pade approximation firstly proposed by Hadley[4]. However, its disadvantage is that when using a difference scheme such as the Crank-Nicolson scheme to solve this high-order equation, a tridaigonal matrix no longer results. Thus a generalized solver routine and complicated program coding are required. In particular complicated manipulations of matrix equations of bandwidth 2n+1 for a Pade(n,n) approximation will be required. Instead of the higher-order Pade approximations, a new approach for developing the square root operator is to use the Pade series/product involving only first powers [2,3]. We can replace a complicated rational function by a succession of simpler ones. Finally, the solution of the 3D operator splitting method with the Pade series/product expansion are presented for a ridge-type 3D waveguide, a curved 3D waveguide model and a primitive MMI device.

2. General Formalism

We consider the three-dimensional waveguide model. The 3D semi-vector Helmholtz equation for optical field is given by

$$\frac{\partial}{\partial x}\left\{\frac{1}{\varepsilon_r}\frac{\partial}{\partial x}(\varepsilon_r E)\right\}+\frac{\partial^2 E}{\partial y^2}+\frac{\partial^2 E}{\partial z^2}+k_0^2\varepsilon_r E=0 \quad (1)$$

where x,y and z are space variables and range variable, $n^2(x,y,z)=\varepsilon_r(x,y,z)$ is the index of refraction, k_0 is the reference wavenumber of free space.

By assuming the optical wave propagates along the +z direction, the optical field can be separated as a slowly varying envelope and a fast oscillating phase term, i.e., substituting a solution of the form $E(x,y,z)=\phi(x,y,z)\exp(-jk_0n_bz)$ into Eq.(1) and factoring, the wave equation is transformed into the following equation for the slowly varying complex amplitude $\phi(x,y,z)$:

$$\frac{\partial\phi}{\partial z}=-jk_b\sqrt{1+X+Y}\ \phi, \quad (2)$$

where

$$X\equiv k_b^{-2}\left(\frac{\partial}{\partial x}\left\{\frac{1}{\varepsilon_r}\frac{\partial}{\partial x}(\varepsilon_r\phi)\right\}+\frac{1}{2}\left(k^2-k_b^2\right)\right),$$

$$Y\equiv k_b^{-2}\left(\frac{\partial^2\phi}{\partial y^2}+\frac{1}{2}\left(k^2-k_b^2\right)\right),$$

We suppose now that

$$\sqrt{1+X+Y}\,\frac{\partial\phi}{\partial z}=\frac{\partial}{\partial z}\sqrt{1+X+Y}\ \phi$$

Then, if above relation holds, the approximation equation leads to a 3D-PEM that offers an attractive combination of accuracy and efficiency[2]

$$\sqrt{1+X+Y}\cong 1+\sum_{i=1}^{n}\frac{a_{i,n}X}{1+b_{i,n}X}+\frac{1}{2}Y \quad (3)$$

By applying Eq.(3) to approximate the square root in Eq.(2), the one-way wave equation in the + z-propagation direction can be reduced to the wide-angle 3D parabolic equation,

$$\frac{\partial\phi}{\partial z}=-jk_b\left(1+\sum_{i=1}^{n}\frac{a_{i,n}X}{1+b_{i,n}X}+\frac{1}{2}Y\right)\phi, \quad (4)$$

$$a_{i,n} = \frac{2}{2n+1}\sin^2\frac{i\pi}{2n+1}, \qquad b_{i,n} = \cos^2\frac{i\pi}{2n+1}$$

For the sake of simplicity, Eq.(4) can be also written symbolically as follows:

$$\frac{\partial\phi}{\partial z} = L_1\phi + L_2\phi + \cdots + L_N\phi \quad (i=1,2,....,N) \qquad (5)$$

where the terms L_i represent the linear operator of Eq.(4). One form of operator splitting would be to get from n to n+1 by the following sequence of updatings:

$$\left(I - \frac{\Delta z}{2}L_i\right)\phi^{n+i/N} = \left(I + \frac{\Delta z}{2}L_i\right)\phi^{n+(i-1)/N} \qquad (6)$$

When Eq.(6) is discretized with the difference scheme of Crank-Nicolson, the truncation error in approximating the second-order differential in the transverse direction is of second-order $O(\Delta x^2)$. To reduce this truncation error, the partial differential involving in the operator, defined by formula (2), is replaced by the Douglas operator considering the 4th-order differential. Furthermore, Varga's treatment is applied to handle the dielectric discontinuity on the boundary interface between core and cladding [5-8].

$$\frac{\partial}{\partial x}\left(\frac{1}{n^2}\left\{\frac{\partial}{\partial x}\left(n^2\phi\right)\right\}\right) = \frac{1}{\Delta x^2}\frac{\delta_x^2}{1+\alpha\delta_x^2}\ , \qquad (7)$$

$$\alpha = \begin{cases} 0 & \text{(Crank-Nicolson)} \\ 1/12 & \text{(Douglas-scheme)} \end{cases}$$

where $\left(\delta_x^2\phi\right)_i = A_{i-1}\phi_{i-1} - B_i\phi_i + C_{i+1}\phi_{i+1}$,

$$A_{i-1} = \frac{2n_{i-1}^2}{n_{i-1}^2 + n_i^2}\ ,\ C_{i-1} = \frac{2n_{i+1}^2}{n_{i+1}^2 + n_i^2}\ , B_i = A_{i-1} + C_{i+1}$$

When the Douglas operator is applied, we can easily obtain the high accuracy six-point scheme and a tridiagonal system of complex linear equations. The present scheme allows us to use an efficient procedure such as the Thomas algorithm or LU decomposition, so that the computational time is almost identical to that in the conventional PE method based on the Crank-Nicolson scheme.

3. Numerical Results

In order to check the validity and limitation of our developed 3D parabolic equation method, we simulated benchmark tests for which exact or other numerical solutions are obtained. The waveguiding structures are shown in Fig.1. The step-size in the cross section is $\Delta x=\Delta y=0.1[\mu m]$, and the propagation-increment is $\Delta z=0.25[\mu m]$. To avoid the effect on optical fields by the reflection from the calculation window edge, a transparent boundary condition is implemented at the edge of the cross section for the program. In the first example propagation of the ridge-type 3D waveguide excited with a focused Gaussian beam at the input is investigated. As shown in Fig.1(a), the refractive indexes are n_a=1, n_f=3.44 and n_s=3.40. The wavelength $\lambda$is 1.15 μm,a wave incident in waveguide 1 is gaussian beam and the width W is 3 μm. The steady-state field distribution is observed at the propagation distance z=1000μm. It is found that the electric field transfers from one side to the other side at the propagation distance z=4000μm. As our second example, we consider a primitive Multi-Mode Interference coupler reported in [9] as shown in Fig.1(b) . The 13.28μm wide multimode section supports 13 guided modes and has length L=2312μm. Figure 3 shows the simulation result of 2x2 MMI coupler. It is found that direct and mirror single images of the input field are occur because of interference at even and odd multiples of the length ($3L\pi$). Finally, we simulated a 90 degree bent waveguide as shown in Fig.1(c). Figure 4 shows a contour line display of the optical field distribution for this bent strip waveguide.Here R= 200 μm is the radius of curvature, λ=0.6328 μm,W=3.0 μm and n_a=1, n_f=1.491, n_s=1.46 . It is found that the initial Gaussian field deviates from the center of bent waveguide.

4. Conclusion

The recently developed finite-difference parabolic equation method combined a Douglas operator scheme to the Pade series expansion is given for the field propagation properties of waveguides and applied in the numerical analysis of benchmark tests. The numerical results for Pade(2,2) approximation is almost the same as the exact field pattern, although the Pade (1,1) approximation fails to give good results. Since the sum form of the linear rational function is more sensitive to round off errors than the product form, it is necessary to program in double precision. In the near future, the algorithms proposed here will be applied and extended to optical field propagation in practical longitudinally varying 3D dielectric planar circuits.

[Reference]

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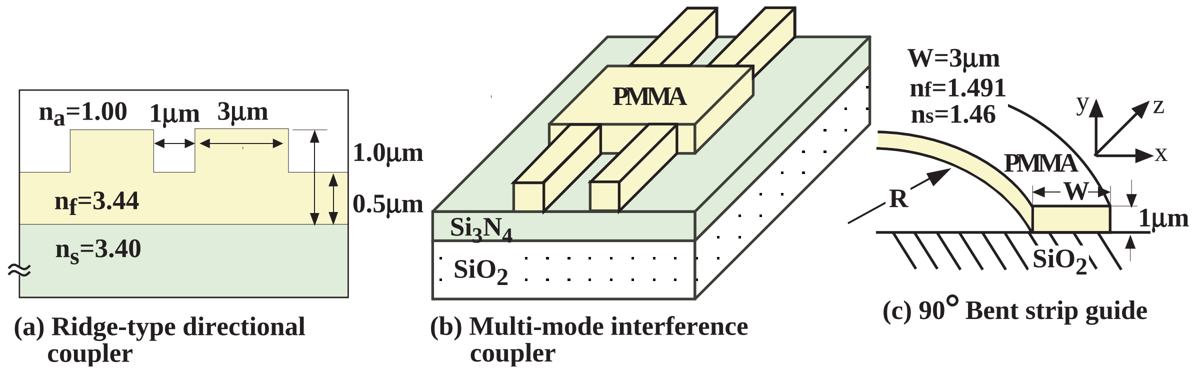


Fig.1 Strucure and model of 3D optical waveguides.

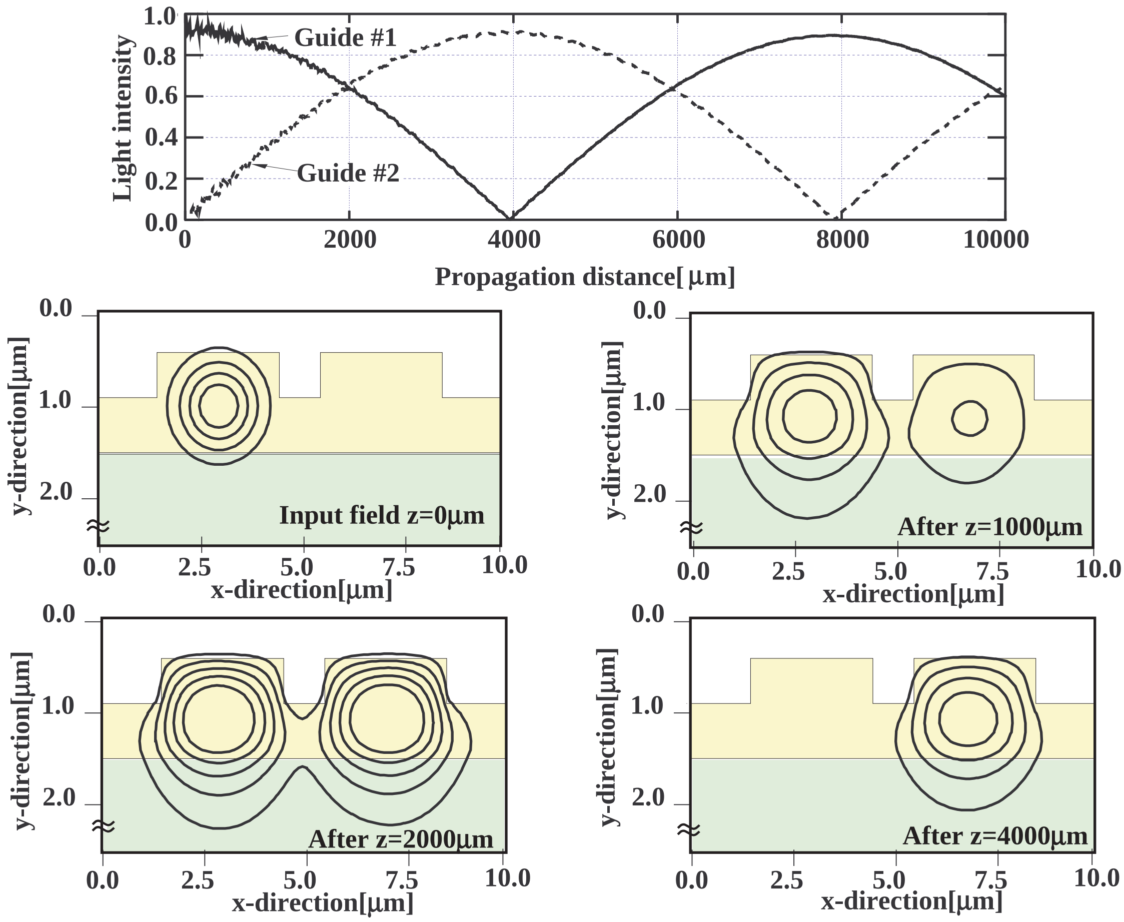
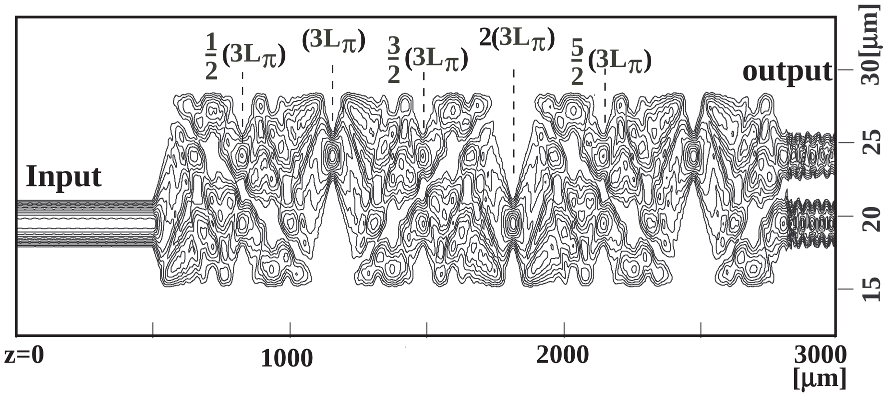


Fig.2 Optical field distribution of ridge-type 3D waveguide by contour display.



$L_\pi = \dfrac{\pi}{\beta_0 - \beta_1}$:

L_π : Beat length of two lower mode

W_{in} and W_{out} : 3 μm

W_M and length: 13.28 μm, 2312 μm.

Wavelength: 0.6328 μm

n_{core}=1.63066, n_{clad}=1.5918

Fig.3 Optical intensity pattern of MMI coupler.

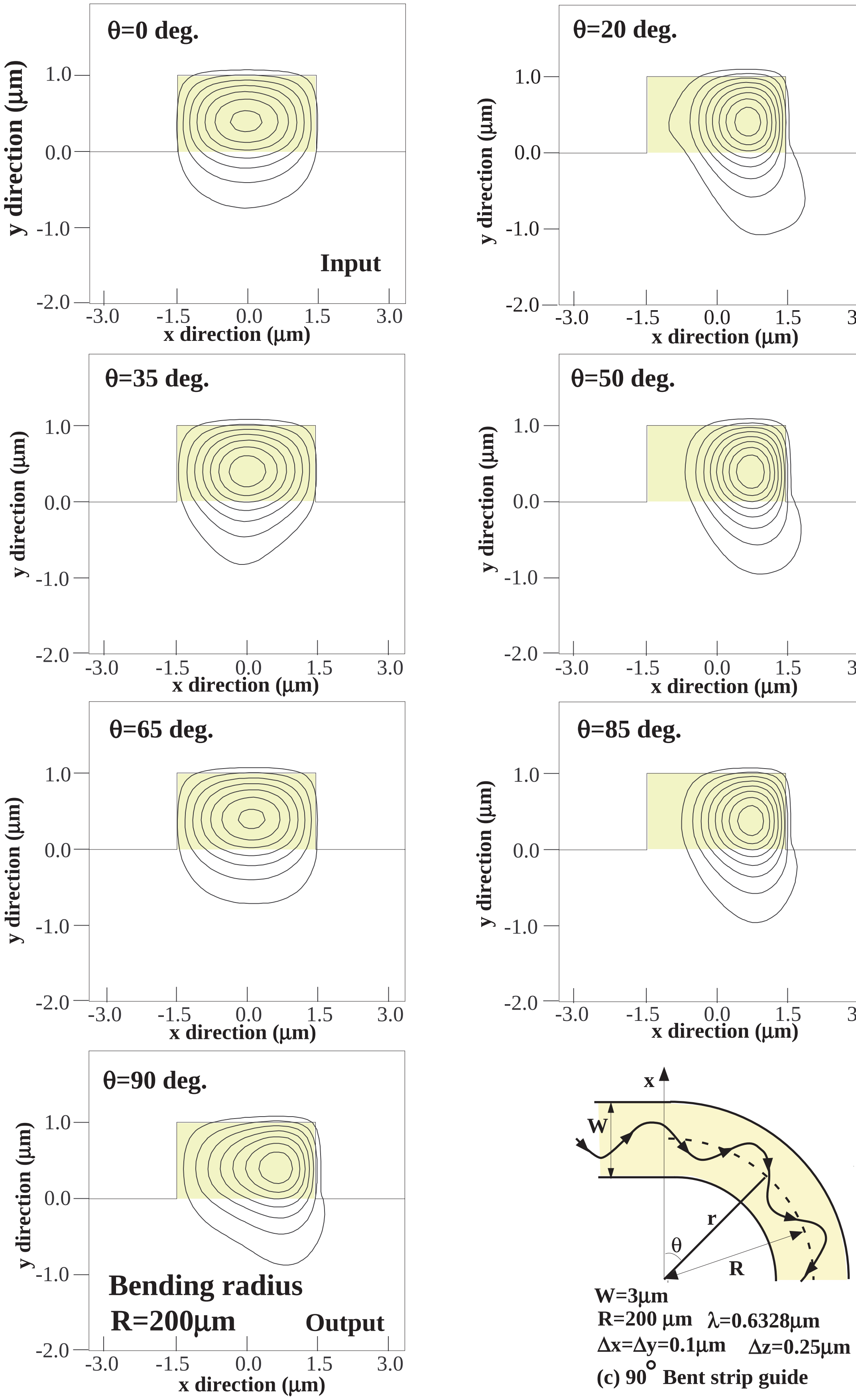


Fig.4 Field propagation for bent strip waveguide as a function of bend angle z=Rθ.